Notation:

All quantities with an asterisk all in the c.m. and all other quantities (if not invariants) ar in the lab system. Subscripts used for projectile, target, and final state particles are p, t, 1, and 2, respectively.

T_p	projektile kinetic energy	s	total invariant mass
p_p	projectile lab momentum	t	four momentum transfer
θ_i	azimuthal scattering angle	ϕ	polar angle of final state particles
$\mathcal{T}_i$	Time of flight	z_s	intercept in slab layer
z_r	intercept in ring layer		where particles strikes detector element
R_s	Radius of slab-layer	R_r	radius of semiring-layer
E_i	*total* energy of particle i	T_i	*kinetic* energy of particle i
p_i	momentum of particle i	P_i	four-momentum of particle i
$p_{\|\|}$	longitudinal momentum component	$p_\perp$	transverse momentum compontent
E^*	c.m. energy of all particles	p^*	c.m. momentum of all particles
$\gamma_{\mathrm{c.m.}}$	Lorentz parameter lab to c.m.	$\beta_{\mathrm{c.m.}}$	Lorentz parameter lab to c.m.
γ_i	Lorentz parameter for particle i	β_i	Lorentz parameter for particle i
m_p	proton mass		

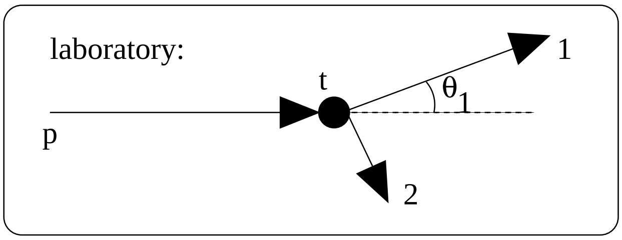
A useful function is the "triangle function"

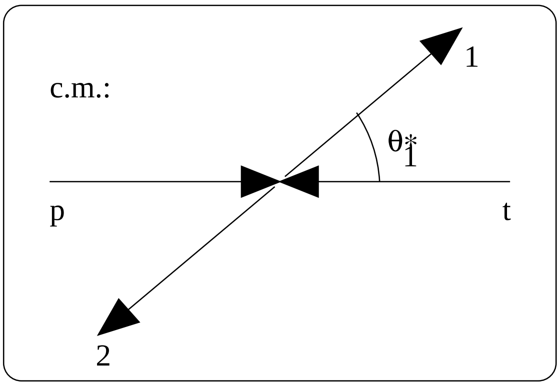
$$\lambda(a,b,c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \qquad (1)$$

Four momenta are defined as:

$$P_i = (E_i, \vec{p_i}) \;\; ; \qquad P_1 P_2 = E_1 E_2 - \vec{p_1} \cdot \vec{p_2} \qquad (2)$$

Throughout, units are used, where $c = \hbar = 1$





Some useful formulaes for pp elastic scattering:

Conversion: beam momentum-kinetic energy:

$$p_p = \sqrt{T_p(2m_p + T_p)} \tag{3}$$

Invariants:

$$s = (P_p + P_t)^2 = (P_1 + P_2)^2 = 2m_p(2m_p + T_p) \tag{4}$$

$$t = (P_p - P_1)^2 = (P_2 - P_t)^2 = 2\left\{m_p^2 - (E^*)^2 + (p^*)^2 \cos(\theta_1^*)\right\} = m_p T_p \left\{\cos(\theta_1^*) - 1\right\} \tag{5}$$

Lorentz-boost parameters:

$$\gamma_{\text{c.m.}} = \frac{\sqrt{s}}{2m_p} = \sqrt{1 + \frac{T_p}{2m_p}} \tag{6}$$

$$\beta_{\text{c.m.}} = \sqrt{1 - \frac{4m_p^2}{s}} = \sqrt{1 + \frac{2m_p}{T_p}}^{-1} \tag{7}$$

$$\beta_{\text{c.m.}}\gamma_{\text{c.m.}} = \sqrt{\frac{s}{4m_p^2} - 1} = \sqrt{\frac{T_p}{2m_p}} \tag{8}$$

c.m. quantities:

$$E^* = \sqrt{s}/2 = m_p\sqrt{1 + \frac{T_p}{2m_p}} \tag{9}$$

$$p^* = \frac{\sqrt{\lambda(s, m_p^2, m_p^2)}}{2\sqrt{s}} = \frac{\sqrt{s - 4m_p^2}}{2} = \sqrt{\frac{m_p T_p}{2}} = \frac{p_p m_p}{\sqrt{s}} \tag{10}$$

lab quantities:

$$p_p = \frac{\sqrt{\lambda(s, m_p^2, m_p^2)}}{2m_p} = \sqrt{s\left(\frac{s}{4m_p^2} - 1\right)} \tag{11}$$

$$E_1 = \frac{s + t - 2m_p^2}{2m_p} = \frac{s}{4m_p} + \left(\frac{s}{4m_p} - m_p\right)\cos(\theta_1^*) = m_p + \frac{T_p}{2}\left(1 + \cos(\theta_1^*)\right) \tag{12}$$

$$E_2 = \frac{2m_p^2 - t}{2m_p} = \frac{s}{4m_p} - \left(\frac{s}{4m_p} - m_p\right)\cos(\theta_1^*) = m_p + \frac{T_p}{2}\left(1 - \cos(\theta_1^*)\right) \tag{13}$$

$$T_i = \frac{T_p}{1 + {\gamma_{\text{c.m.}}}^2 \tan^2(\theta_i)} \quad ; \quad i = 1, 2 \tag{14}$$

Symmetric angle:

$$\theta_{sym} = \arctan\frac{1}{\gamma_{\text{c.m.}}} \tag{15}$$

$$\mathcal{T}_{j,i} = \frac{\sqrt{R_j^2 + z_j^2}}{\beta_i} = \frac{R_j E_i}{p_{\perp i}} = \frac{R_j E_i}{p^* \sin(\theta_1^*)} \; ; \quad i = 1, 2; \; j = s, r \tag{16}$$

lab to c.m. conversion:

$$\tan(\theta_i) = \frac{\sin(\theta_i^*)}{\gamma_{\text{c.m.}}\left\{\cos(\theta_i^*)+1\right\}} \;; \qquad i=1,2 \tag{17}$$

$$\cos(\theta_i^*) = \frac{1-\gamma_{\text{c.m.}}{}^2\tan^2(\theta_i)}{1+\gamma_{\text{c.m.}}{}^2\tan^2(\theta_i)} \tag{18}$$

$$\tan\frac{\theta_i^*}{2} = \gamma_{\text{c.m.}}\tan\theta_i \tag{19}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}\Omega^*} = \frac{\left\{1+\gamma_{\text{c.m.}}{}^2\tan^2(\theta_i)\right\}^2\cos^3(\theta_i)}{4\;\gamma_{\text{c.m.}}{}^2} \tag{20}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}\Omega^*} = \frac{\gamma_{\text{c.m.}}\left\{1+\tan^2(\theta_i^*/2)\right\}^2}{4\sqrt{\gamma_{\text{c.m.}}{}^2+\tan^2(\theta_i^*/2)}^3} \tag{21}$$

Relations implied by pp-kinematics:

$$\mid \phi_1 - \phi_2 \mid = \pi \tag{22}$$

$$\tan(\theta_1)\tan(\theta_2) = \frac{R_j^2}{z_{j,1}z_{j,2}} = \gamma_{\text{c.m.}}{}^{-2}; \quad j=s,r \tag{23}$$

$$z_{j,1}+z_{j,2} = \frac{2R_j\gamma_{\text{c.m.}}}{\sin(\theta_1^*)}; \quad j=s,r \tag{24}$$

$$z_{j,1}-z_{j,2} = \frac{2R_j\gamma_{\text{c.m.}}}{\tan(\theta_1^*)}; \quad j=s,r \tag{25}$$

$$\mathcal{T}_{j,1}+\mathcal{T}_{j,2} = \frac{R_j\ s}{m_p\sqrt{s-4m_p^2}\sin(\theta_1^*)} \;; \quad j=s,r \tag{26}$$

$$\mathcal{T}_{j,1}-\mathcal{T}_{j,2} = \sqrt{s-4m_p^2}\frac{R_j}{m_p\tan(\theta_1^*)} \;; \quad j=s,r \tag{27}$$

$$\frac{\mathcal{T}_{k,1}-\mathcal{T}_{k,2}}{z_{j,1}-z_{j,2}} = \frac{R_k}{R_j}\sqrt{1-\frac{4m_p^2}{s}} \;; \quad k,j=s,r \tag{28}$$